IMAFF-RCA-05-07

On the accretion of phantom energy onto wormholes

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(Dated: February 5, 2008)

By using a properly generalized accretion formalism it is argued that the accretion of phantom energy onto a wormhole does not make the size of the wormhole throat to comovingly scale with the scale factor of the universe, but instead induces an increase of that size so big that the wormhole can engulf the universe itself before it reaches the big rip singularity, at least relative to an asymptotic observer.

PACS numbers: 98.80.Hw, 04.70.-s

Keywords: Lorentzian Wormholes, Dark Energy, Accretion

Some attention has been recently paid to the consideration of cosmic systems formed by wormholes and phantom energy [1,2]. In particular, it appears of great interest the study of dark energy accretion onto gravitating systems, such as black holes [3], wiggly cosmic strings [4] and wormholes [5,6]. In the latter case it was first claimed [5] that accretion of phantom energy would induce an increase of the wormhole throat radius so quick that the wormhole would engulf the entire universe before this reached the big rip singularity. Such a result has been dubbed big trip and was later criticized by Faraoni and Israel [6] who have used several cases of exotic-matter shell on the throat, always obtaining the conclusion that the wormhole becomes asymptotically comoving with the cosmic fluid so that the future evolution of the universe keeps being causal. In this letter I will argue that the result derived by Faraoni and Israel cannot be recovered if one uses a proper accretion theory for dark energy onto wormholes. Actually, the result of Ref. [6] would just describe what one should expect when the inflationary effects of the accelerated expansion of the universe on the wormhole size are taken into account. In fact, when just such effects are considered it is obtained that wormholes inflate so that their size would scale like the scale factor of the universe [7], similarly to what later was obtained by Faraoni and Israel [6].

A proper dark-energy accretion model for wormholes should be obtained by generalizing the Michel theory [8] to the case of wormholes. Such a generalization has been already performed by Babichev, Dokuchaev and Eroshenko [3] for the case of dark-energy accretion onto Schwarzschild black holes. Here I will adapt the same procedure to the dark-energy accretion onto the Morris-Thorne wormhole case. The most general static space-time metric of one such wormholes is given by [9]

$$ds^2 = -e^{\Phi(r)}dt^2 + \frac{dr^2}{1-\frac{K(r)}{r}} + r^2 d\Omega_2^2, \tag{1}$$

where the shift function $\Phi(r)$ can be taken to be either zero or a given function of the radial coordinate r, the shape function $K(r)$ can be taken either as $K(r) = K_0^2/r$ for wormholes with zero tidal force, or as $K(r) = K_0[1-(r-K_0)/R_0]^2$ if the exotic matter is confined into an arbitrarily small region with width R_0 around the wormhole throat, and $d\Omega_2^2$ is the metric on the unit two-sphere.

We shall follow now the procedure used by Babichev, Dokuchaev and Eroshenko [3], adapting it to the case of a Morris-Thorne wormhole, and thus consider the tensor momentum-energy for a perfect fluid

$$T_{\mu\nu} = (p+\rho)u_\mu u_\nu + p g_{\mu\nu}, \tag{2}$$

in which p is pressure, ρ is energy density and $u^\mu = dx^\mu/ds$ is the four-velocity. By integrating then the time component of the conservation law for momentum-energy tensor, $T^\nu_{\mu;\nu} = 0$, we can obtain for the spherical symmetry of our metric,

$$um^{-2}r^2\left(1-\frac{K(r)}{r}\right)^{-1}(p+\rho)\sqrt{u^2+\frac{K(r)}{r}-1} = C, \tag{3}$$

where m is the exotic mass of the wormhole which, following the procedure of Ref. [3], has been introduced to render the integration constant C to have the dimensions of an energy density (note that we are using natural units so that $G = c = \hbar = 1$), $u = dr/ds$ and, without any loss of generality for our present purposes, we have adhered to the case where $\Phi = 0$. We now also integrate the projection of the conservation law for momentum-energy tensor onto the four-velocity, $u_\mu T^{\mu\nu}_{;\nu} = 0$. For a perfect fluid and spherical symmetry, since $u > 0$, we finally obtain

$$m^{-2}r^2 u\left(1-\frac{K(r)}{r}\right)^{-1/2} e^{\int_{\rho_\infty}^{\rho}\frac{d\rho}{p+\rho}} = A, \tag{4}$$

with A a positive dimensionless integration constant. From Eqs. (3) and (4) we get finally

$$\left(1-\frac{K(r)}{r}\right)^{-1/2}\sqrt{u^2+\frac{K(r)}{r}-1}(p+\rho)e^{-\int_{\rho_\infty}^{\rho}\frac{d\rho}{p+\rho}} = B, \tag{5}$$

in which $B = C/A = \hat{A}\left(\rho_\infty + p(\rho_\infty)\right)$, with $\hat{A}$ a positive constant.

The most general expression for the rate of change of the exotic wormhole mass should be given by integrating the momentum density T_0^r over the element of two dimensional spherical surface $dS = r^2\sin\theta d\theta d\phi$ [10], i.e.

$$\dot{m} = \int dS T_0^r, \tag{6}$$

where the sign has been chosen so that the rate refers to a negative energy. We then obtain from Eqs. (4) and (5),

$$\dot{m} = -4\pi m^2 Q\sqrt{1-\frac{K(r)}{r}}(p+\rho), \tag{7}$$

with the constant $Q = A\hat{A} > 0$. For the relevant asymptotic regime $r \to \infty$, the rate $\dot{m}$ reduces to

$$\dot{m} = -4\pi m^2 Q(p+\rho). \tag{8}$$

We see then that the rate for the wormhole exotic mass due to accretion of dark energy becomes exactly the negative to the similar rate in the case of a Schwarzschild black hole, asymptotically. It should be pointed out however that it is not a necessary condition that the wormhole possesses negative energy densities (as measured by static observers), but that it violates the null energy condition. Had we considered a positive sign in Eq. (7) and a positive wormhole energy density (and positive ADM mass), the respective accretion of phantom energy had been accompanied with a diminishing of the wormhole mass, as in the case of black holes analyzed by Babichev, Dokuchaev and Eroshenko in Ref. [3].

For a quintessence model with equation of state $p = w\rho = w\rho_0 a^{-3(1+w)}$ and scale factor

$$a = T^{2/[3(1+w)]} = \left(a_0^{3(1+w)/2} + \frac{3}{2}(1+w)C(t-t_0)\right)^{2/[3(1+w)]}, \tag{9}$$

with $C = \sqrt{8\pi G\rho_0/3}$, one can integrate Eqs. (7) and (8). For the phantom regime $w < -1$, we obtain from the asymptotic expression (8) the evolution of the exotic mass of the wormhole,

$$m = \frac{m_0}{1 - \frac{4\pi Q\rho_0 m_0(|w|-1)(t-t_0)}{T(w<-1)}}, \tag{10}$$

where m_0 is the initial exotic mass of the wormhole. This result is consistent with what was obtained in Ref. [5], implying that the exotic mass m diverges at the time

$$t_* = t_0 + \frac{t_{br} - t_0}{1 + \frac{8\pi Q m_0 a_0^{3(|w|-1)/2}}{3C}}, \tag{11}$$

where t_{br} is the time at which the big rip takes place, i.e. $t_{br} = t_0 + 2a_0^{-3(|w|-1)/2}/[3(|w|-1)C]$. Thus, t_* occurs before the big rip. Now, depending on the distribution of the exotic mass on the throat, we can derive the variation of the wormhole throat radius with phantom energy accretion for the asymptotic case. In particular, for a spherical shell distribution with constant thickness, the wormhole throat radius K_0will be given by

$$K_0 = \frac{K_{0i}}{\sqrt{1 - \frac{4\pi Q\rho_0 m_0(|w|-1)(t-t_0)}{T(w<-1)}}}, \tag{12}$$

with K_{0i} the initial size of the wormhole throat. In this case the wormhole can only evolve until the time t_*. If we take $K_0 \propto m$, then we would get the same result as derived in Ref. [5] for the wormhole throat, this time without assuming any relation between the sizes of the wormhole and the corresponding Schwarzschild black hole. In any event, if we let $r \to \infty$, the size of the wormhole would exceed the size of the universe before this reached the big rip singularity, so restoring the problem with causality in the future of the universe.

Thus, by applying a proper accretion dark energy theory to the wormhole, we obtain that, superposed to the inflationary effects that the universal acceleration has on the wormhole size which thereby scales like the scale factor [6,7], there is a real income of dark energy which adds to the mass of the wormhole when $w < -1$. In the asymptotic case $r \to \infty$, big trips violating causality could then be performed by the universe as a whole if the equation-of-state parameter of the universe would keep on a constant value less than -1 in the future.

The above conclusion is only strictly valid if we allow $r \to \infty$. For otherwise, if we assume e.g. $K_0^2 = \gamma m$, with γ a constant, for wormholes with zero tidal forces, the integration of Eq. (7) yields

$$\left[\frac{\sqrt{1-\frac{\gamma m}{r^2}}}{m} + \frac{\gamma}{2r^2}\ln\left(\frac{2-\frac{\gamma m}{r^2}+2\sqrt{1-\frac{\gamma m}{r^2}}}{m}\right)\right]\Bigg|_{m_0}^{m} = -\frac{4\pi Q\rho_0(1+w)(t-t_0)}{a_0^{3(1+w)/2}T}. \tag{13}$$

It can be checked that the accretion of phantom energy in this case also induces an increase of the wormhole exotic mass, but that increase can just be produced until m reaches a maximum value given by $m \le m_{max} = r^2/\gamma$.

I consider the conclusions drawn in the present letter to be weird but still correct. It could be at first sight argued that since metric (1) is static our analysis can only be a good approximation for very small accretion rates. However, even though the back reaction on the wormhole metric (1) has not been explicitly considered, the general conclusions of our study must still remain valid at any accretion rate and hence for the big trip case, as should be Eqs. (3) and (4), because the modifications that the use of a time-dependent wormhole metric induced in these expressions are all expected to vanish on the asymptotic limit where the big trip feature should occur, at least if all time-dependence in the metric is confined to take place only in the throat radius so that the shape of the wormhole is preserved during phantom energy accretion. In this case, the use of a non static wormhole metric would lead to significant changes in all of our calculations for $r < \infty$, but leave the calculation given in this letter essentially unchanged and valid at any accretion rate asymptotically.

Acknowledgments

This work was supported by MCYT under Research Project No. BMF2002-03758.

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